RR photons

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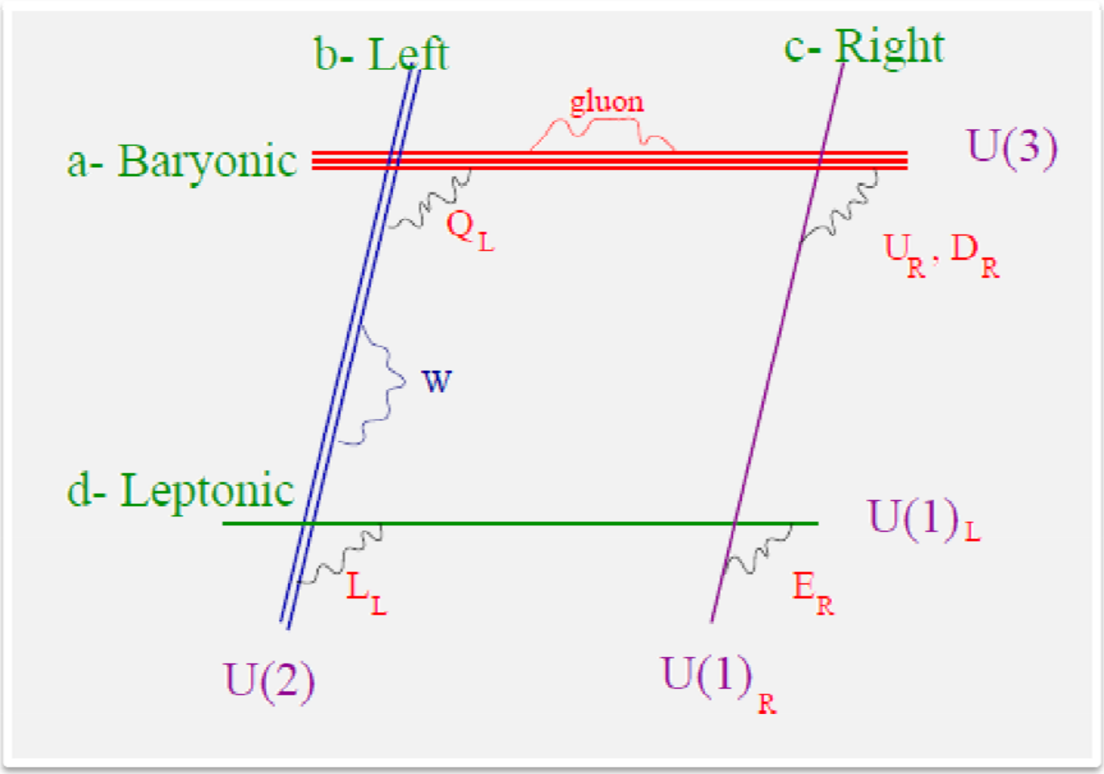
Based on P.G.C, L. E. Ibáñez and F. Marchesano, **JHEP 1109 (2011) 110**
arXiv:**1106.0060** [hep-th]

Iberian Strings '12, Bilbao, 31 January - 2 February 2012

1. Motivation

Semi-realistic string theory compactifications generically lead to U(1) gauge symmetries beyond $U(1)_Y$



[Cremades, Ibanez, Marchesano ’02]

1. Motivation

- Some of these extra U(1)'s acquire masses via the Stückelberg mechanism

$$\mathcal{L} \supset C_2 \wedge F_2 \quad \Rightarrow \quad \mathcal{L}_{\rm Stk} = \frac{1}{2}(d\rho + qA)^2 \qquad (d\rho = *_4 dC_2)$$

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$M_{U(1)} \sim M_s$ ⇒ global symmetries

broken by non-perturbative effects to discrete subgroups (e.g. matter parity, baryon triality)

[Berasaluce et al. '11]

Only detectable at experiments if M_s ~ 1 TeV (WIMPs)

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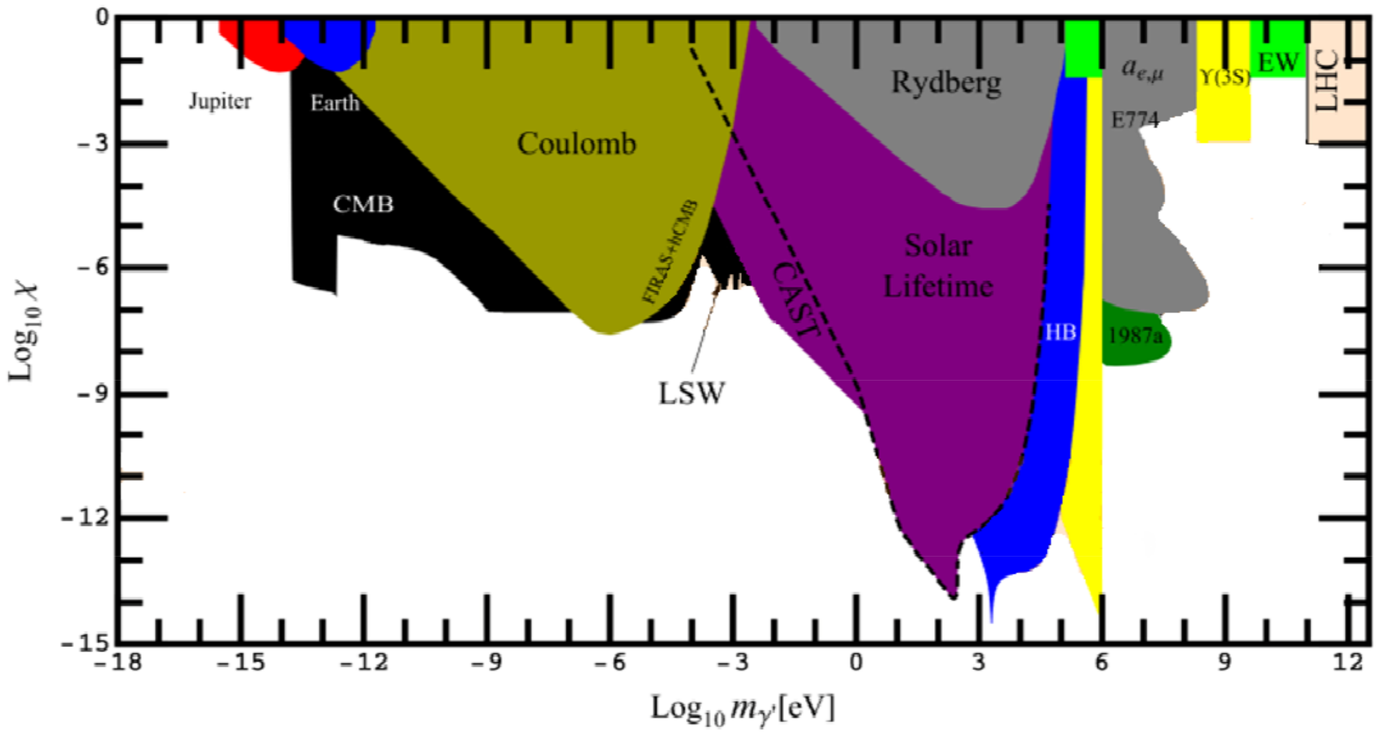
• Other U(1)'s however may remain massless or very light (WISPs) and lead to light hidden U(1) gauge symmetries compatible with experiment.

1. Motivation

Light hidden U(1) gauge symmetries are a window of opportunity to hidden sector physics, even at large string scale

$$\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\chi}{2}X_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_{\gamma'}^2 X_\mu X^\mu$$

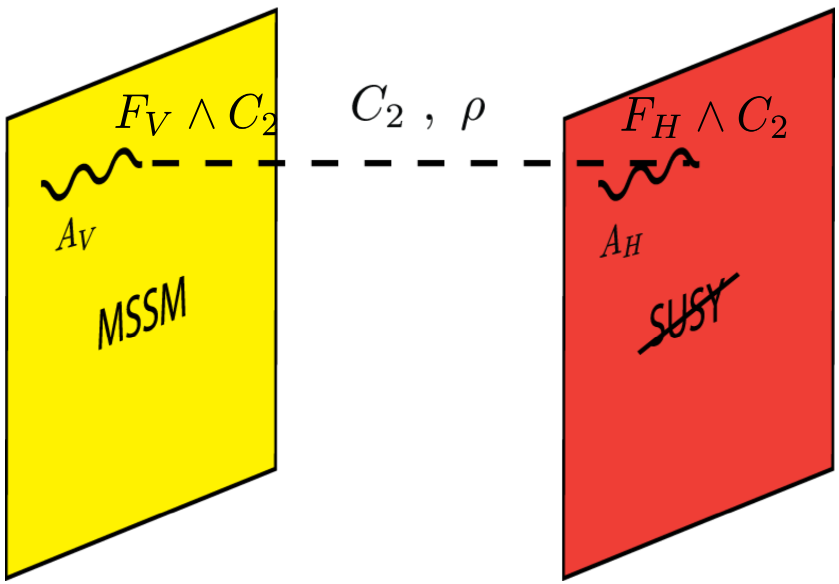


[Jaeckel, Ringwald '10]

1. Motivation

• Hidden U(1)'s are also a possible mechanism for mediating SUSY breaking in a flavor independent way:

[Langacker et al '07]
[Verlinde et al. '07]



$$\mathcal{L} \supset \frac{1}{2}|d\rho + eA_V + qA_H|^2$$

$$U(1)_Y = eU(1)_V - qU(1)_H$$
$$U(1)_X = eU(1)_V + qU(1)_H$$

1. Motivation

In type II string theory compactifications there are two sources of hidden U(1) gauge symmetries:

- D-branes located ‘far away’ from the MSSM D-brane sector

- Bulk U(1)’s arising from KK reduction of the Ramond-Ramond closed string fields ⇨ they are generic and have no massless matter charged under them

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It is therefore natural to ask:

- *Can RR U(1)'s mix with the hypercharge??*
- *If so, can we compute* χ *and* $m_{\gamma'}$ *??*
- *Can we obtain new phenomenological scenarios ??*

2. U(1)'s in type IIA compactifications

Type IIA string theory on a CY orientifold $\mathbb{R}^{1,3} \times \mathcal{M}_6/\Omega_p(-1)^{F_L}\sigma$

$$\sigma J = -J \ , \qquad \sigma\Omega = \overline{\Omega}$$

- Closed string spectrum:

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- Closed string spectrum:

$h_-^{1,1} + h^{1,2} + 1$	*N = 1 chiral multiplets*

Scalar components parametrize compactification moduli space:

$$J_c \equiv B_2 + iJ = \sum T^{\hat{i}} \omega_{\hat{i}} \; , \qquad \Omega_c \equiv C_3 + i\mathrm{Re}(C\Omega) = \sum N^I \alpha_I$$

axions

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$h_+^{1,1}$ *N = 1 vector multiplets*

RR U(1) gauge bosons from the expansions: [Grimm, Louis '04]

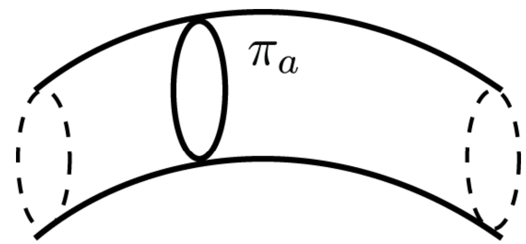
$$C_3 = \sum_I \mathrm{Re}(N^I)\alpha_I + \sum_i A^i \wedge \omega_i \qquad f_{ij} = -i\mathcal{K}_{ij\hat{k}}T^{\hat{k}}$$

2. U(1)'s in type IIA compactifications

D6-brane N = 1 vector & chiral multiplets

D6-branes wrap special Lagrangian 3-cycles in the CY

$$J|_{\pi_a} = 0 \,, \qquad \mathrm{Im}(\Omega)|_{\pi_a} = 0$$



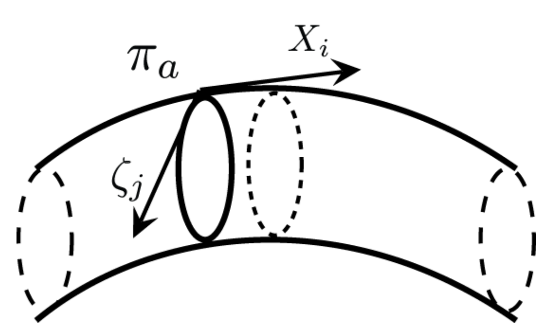
MSSM located in this sector

N_a D6-branes ⇨ $SU(N_a) \times U(1)_a$ $\qquad f_a = -iN_a \int_{\pi_a} \Omega_c$

Deformations preserving sLag parametrized by $b_1(\pi_a)$ adjoint chiral multiplets:

[McLean '98]

$$\Phi_a^j = \theta_a^j + \lambda_i^j \phi_a^i$$



2. U(1)'s in type IIA compactifications

Many D6-brane U(1)'s become massive:

Stückelberg mechanism for D6-brane U(1)'s:

$$\int_{\mathbb{R}^{1,3}\times\pi_a} C_5 \wedge F_2^a = -c_a^I \int_{\mathbb{R}^{1,3}} C_2^I \wedge F_2^a \quad \Rightarrow \quad Q^I = \sum_a c_a^I N_a Q^a$$

massive

$$c_a^I = -\int_{\pi_a} \beta^I$$

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$$c_a^I = -\int_{\pi_a} \beta^I$$

Geometric interpretation: We associate to each D6-brane U(1) an element $\pi_a - \sigma(\pi_a) \in H_3^-(\mathcal{M}_6, \mathbb{R})$

$$Q^b = \sum_a n_a^b Q^a \quad \text{massless} \quad \Leftrightarrow \quad \pi_b^- = \sum_a n_a^b N_a(\pi_a - \sigma(\pi_a)) \quad \text{trivial}$$

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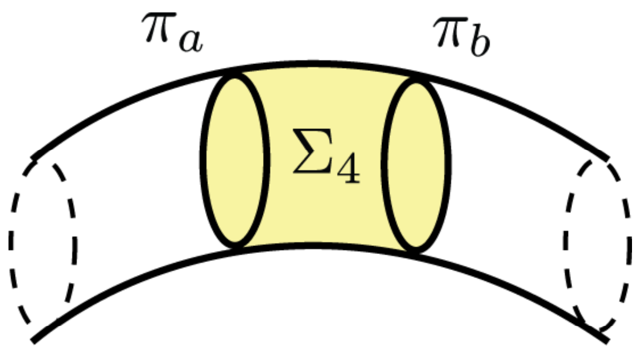
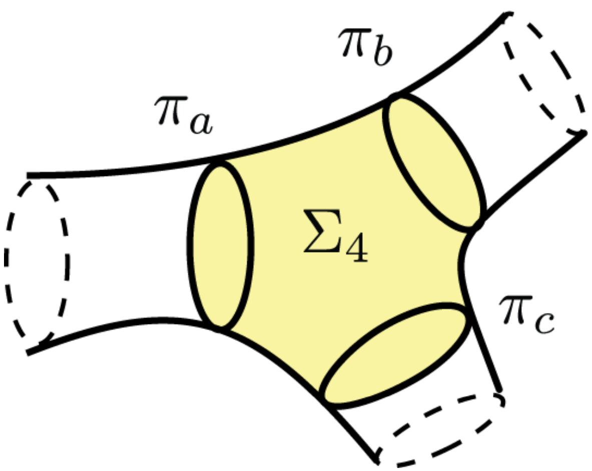


$$\partial\Sigma_4 = \pi_a - \pi_b$$

$$U(2) \to U(1)_a \times U(1)_b$$

$U(1)_a - U(1)_b$ massless

$U(1)_a + U(1)_b$ massive



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3. Kinetic mixing

We have seen the following U(1) charge assigment:

$H_3^-(\mathcal{M}_6, \mathbb{R})$ ⇨ D6-brane U(1)'s

$H_2^+(\mathcal{M}_6, \mathbb{R})$ ⇨ RR U(1)'s

Can D6-brane and RR U(1)'s mix kinematically ??

$$S_{4\mathrm{d,mix}} = -\int_{\mathbb{R}^{1,3}} \left[\mathrm{Re}(f_{ia}) F^i_{\mathrm{RR}} \wedge *_4 F^a_2 + \mathrm{Im}(f_{ia}) F^i_{\mathrm{RR}} \wedge F^a_2\right]$$

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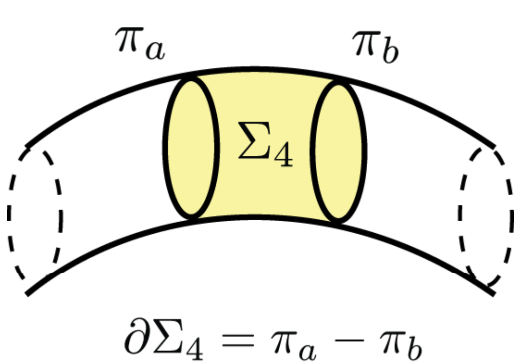
From the D6-brane CS action: $f_{ia} = \Phi_a^j \int_{\pi_a} \omega_i \wedge \zeta^j + \ldots$

Well-defined for massless U(1)'s:



$$f_{i(a-b)} = (\Phi_a^j - \Phi_b^j) \int_{\rho_j} \omega_i + \ldots \quad \Rightarrow \quad f_{ib} = \int_{\Sigma_4} (J_c + F_2^b) \wedge \omega_i$$

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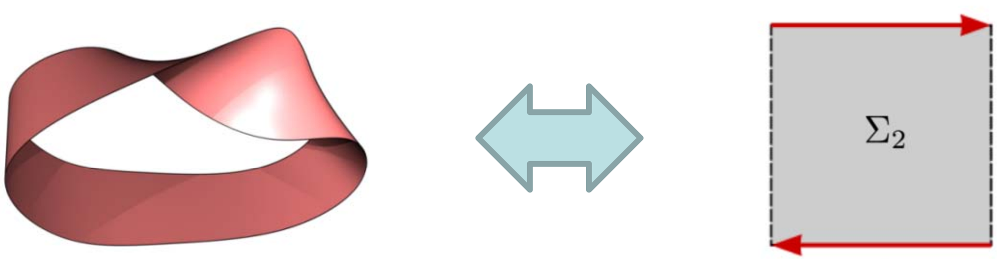
$H_2^+(\mathcal{M}_6, \mathbb{R})$ ⇨ RR U(1)'s

$$H_r(\mathcal{M}_6, \mathbb{Z}) = \underbrace{\mathbb{Z} \oplus \ldots \oplus \mathbb{Z}}_{b_r} \oplus \underbrace{\mathbb{Z}_{k_1} \oplus \ldots \oplus \mathbb{Z}_{k_n}}_{\mathrm{Tor}\ H_r(\mathcal{M}_6)}$$



$$H_1(\mathcal{M}, \mathbb{Z}) = \mathbb{Z}_2$$

$$\partial \Sigma_{r+1} = k \pi_r^{\mathrm{tor}}$$

Does $H_r(\mathcal{M}_6, \mathbb{Z})/H_r(\mathcal{M}_6, \mathbb{R})$ play a role in U(1) physics??

4. Mass mixing

Torsional cycles cannot be detected by 4d massless bulk fields:

$$\int_{\pi_r^{\rm tor}} \omega_r = \frac{1}{k} \int_{\Sigma_{r+1}} d\omega_r = 0$$

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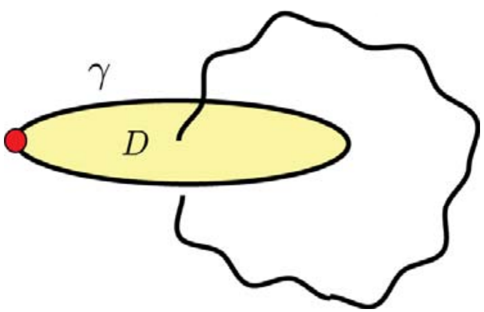
Spectrum of torsional objects constrained by Poincaré duality and UCT:

$$\text{Tor}\, H_3(\mathcal{M}_6, \mathbb{Z}) \simeq \text{Tor}\, H_2(\mathcal{M}_6, \mathbb{Z})$$

$$\text{Tor}\, H_1(\mathcal{M}_6, \mathbb{Z}) \simeq \text{Tor}\, H_4(\mathcal{M}_6, \mathbb{Z})$$



D2-brane wrapping $\pi_2^{\rm tor}$ ⇒ 4d particle

D4-brane wrapping $\pi_3^{\rm tor}$ ⇒ 4d string

$\mathbb{Z}_k$ holonomies:

$$\frac{1}{2\pi i} \log \left[\text{hol}(\gamma, [\pi_2^{\rm tor}]) \right] = L([\pi_2^{\rm tor}], [\pi_3^{\rm tor}]) = \frac{p}{k} \mod 1$$

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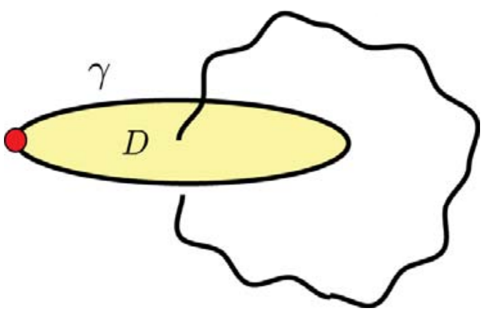

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Linking number

Non-BPS objects in 4d, but stable mod k

Aharanov-Bohm strings and particles [Alford, Krauss, Wilczek ’89]

4. Mass mixing

A-B strings and particles are the consequence of massive U(1)'s higgsed down to a discrete $\mathbb{Z}_k$ gauge symmetry via a Stuckelberg mechanism

[Banks, Seiberg '10]

We can see this more explicitly, by adding the massive forms which correspond to the generators of $\text{Tor } H^4(\mathcal{M}_6) \simeq \text{Tor } H_3(\mathcal{M}_6)$ and $\text{Tor } H^3(\mathcal{M}_6) \simeq \text{Tor } H_2(\mathcal{M}_6)$

$$d\omega_\alpha^{\text{tor}} = k_\alpha{}^\beta \alpha_\beta^{\text{tor}} \ , \quad d\beta^{\text{tor},\beta} = -k^\beta{}_\alpha \tilde{\omega}^{\text{tor},\alpha} \qquad k = L^{-1}$$

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Expanding,

$$C_3 = \sum_\alpha \mathrm{Re}(N^\alpha)\alpha_\alpha^{\mathrm{tor}} + A^\alpha \wedge \omega_\alpha^{\mathrm{tor}} + \dots$$

$$dC_3 = [\mathrm{Re}(dN^\beta) + k^\beta{}_\alpha A^\alpha] \wedge \alpha_\beta^{\mathrm{tor}} + dA^\alpha \wedge \omega_\alpha^{\mathrm{tor}} + \dots$$

Massive RR U(1) gauge symmetries

4. Mass mixing

Massless RR U(1)'s	Massive RR U(1)'s
$H_2^+(\mathcal{M}_6, \mathbb{R})$	$\text{Tor } H_2^+(\mathcal{M}_6, \mathbb{Z})$
Hodge duality: $H_2^+(\mathcal{M}_6, \mathbb{R}) \simeq H_4^-(\mathcal{M}_6, \mathbb{R})$	UCT+Poinc.: $\text{Tor } H_2^+(\mathcal{M}_6, \mathbb{Z}) \simeq \text{Tor } H_3^-(\mathcal{M}_6, \mathbb{Z})$
Intersection number	Linking number
Electric charges: D2 (4d particles)	Electric charges: D2 (4d A-B particles)
Magnetic charges: D4 (4d monopoles)	Magnetic charges: D4 (4d A-B strings)
$U(1)$ gauge symmetry	$\mathbb{Z}_k$ gauge symmetry

$$H_2^+(\mathcal{M}_6, \mathbb{Z})$$

4. Mass mixing

Can D6-brane and RR U(1)'s mix through the Stuckelberg mechanism ??

We have seen that a D4-brane wrapping a torsional 3-cycle

$$[\pi_b^-] = \sum_\beta c_b^\beta [\pi_3^{\text{tor},\beta}]$$

develops a coupling,

$$S_{4d} \supset \sum_\beta c_b^\beta \int_{\mathbb{R}^{1,3}} C_2^\beta \qquad\qquad C_2^\beta \equiv \int_{\pi_3^{\text{tor},\beta}} C_5$$

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Similarly, a D6-brane wrapping the same 3-cycle develops a Stuckelberg coupling in its worldvolume,

$$-\sum_\beta c_b^\beta \int_{\mathbb{R}^{1,3}} C_2^\beta \wedge F_2^b$$

Therefore, massive RR U(1)'s couple to the same complex structure axions than D6-branes do.

4. Mass mixing

Massive RR U(1)'s therefore may mix with D6-brane U(1)'s.

Each linear combination of D6-brane and torsional RR U(1) gauge symmetries has an element of $H_3^-(\mathcal{M}_6, \mathbb{Z})$ associated to it. Massless combinations of U(1)'s are trivial elements in *integer* homology.

$$Q_0 = \sum_a n_a Q^a + \sum_\alpha \check{n}_\alpha Q^\alpha_{RR} \quad \text{massless}$$

$$\Updownarrow$$

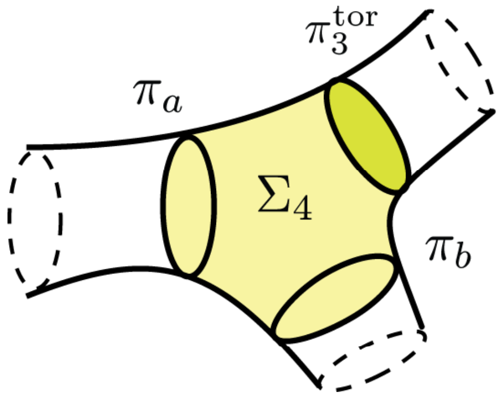
$$\sum_a \frac{N_a n_a}{2}([\pi_a] - [\pi_a^*]) + \sum_{\alpha,\gamma} \check{n}_\alpha k^\alpha{}_\gamma [\pi_3^{\mathrm{tor},\gamma}] = 0$$

Elements which are also trivial in de Rham do not mix with RR U(1)'s

5. Some phenomenological implications

RR U(1)'s allow for new phenomenological scenarios:

- Two stacks of fractional D6-branes which differ by $\pi_3^{\rm tor}$



Massless:

$$U(1)_Y \sim 2U(1)_a - 2U(1)_b + U(1)_{\rm RR}$$

$$f_{YG_2} = -\frac{4i}{27}\sqrt{\frac{10}{3}}(N^0 - T^{\hat{1}})$$

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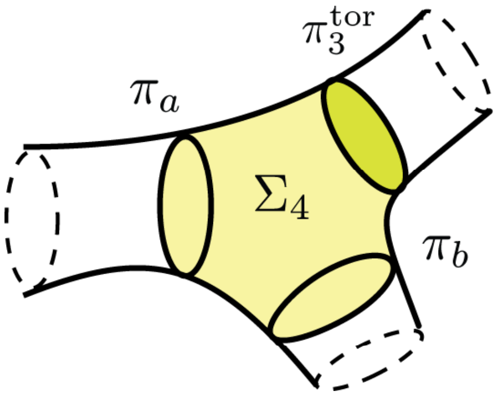

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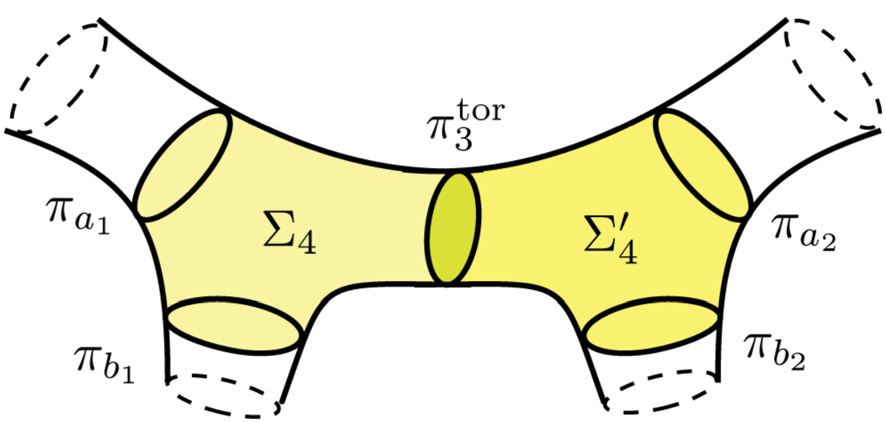
- Two mutualy hidden brane sectors which comunicate via RR photons



Massless:

$$U(1)_{Y_k} \sim 2U(1)_{a_k} - 2U(1)_{b_k} + U(1)_{\rm RR}$$

$$f_{Y_1Y_2} \neq 0$$

5. Some phenomenological implications

RR U(1)'s may also play a role in the context of F-theory GUT's:

[Beasley, Heckman, Vafa '08]
[Donagi, Wijnholt '08]

Hypercharge flux breaking

$$SU(5) \to SU(3) \times SU(2) \times U(1)$$

- 2-cycle ρ^Y trivial in de Rham in order $U(1)_Y$ to remain massless

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If ρ^Y is a torsional 2-cycle of the CY_3

⇓

Mixing of the "hypercharge" with a $U(1)_{RR}$

$$\frac{1}{\alpha_1} = \frac{3}{5\alpha_{SU(5)}} + \frac{k_Y^2}{k_{\rm RR}^2 \alpha_{\rm RR}}$$

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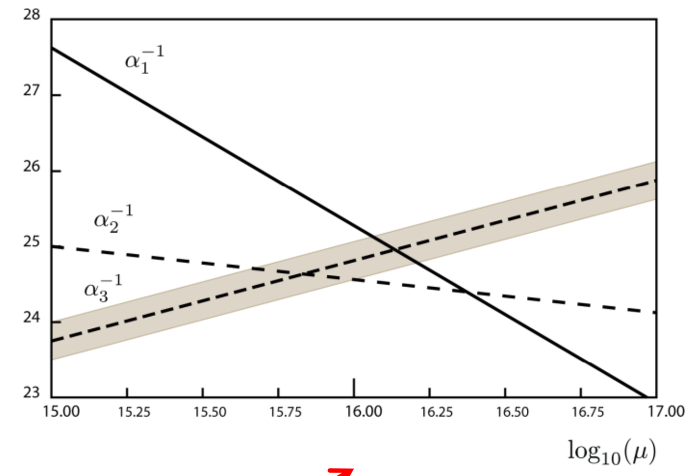

Could explain the known few percent discrepancy in MSSM gauge coupling unification.

6. Torsion and discrete gauge symmetries

One interesting aspect of our analysis is the relation

Discrete gauge symmetries $\Longleftrightarrow$ $\text{Tor}\ H_*(\mathcal{M}, \mathbb{Z})$

which is rather universal: in general one can relate a set of discrete gauge symmetries with a torsion group by considering the corresponding A-B strings and particles.

For instance, for type IIA orientifolds:

$\text{U}(1)_{elec.}$	group	charged particles	cycle	axions	group
${g^m}_\mu$	$\widehat{\text{Tor}}\ H^1_+$	P	$\text{Tor}\ H_1^+$	g_{ij}	$\text{Tor}\ H^2_+$
${B^m}_\mu$	$\widehat{\text{Tor}}\ H^1_-$	$F1$	$\text{Tor}\ H_1^-$	B_{ij}	$\text{Tor}\ H^2_-$
${C_\mu}^{mn}$	$\widehat{\text{Tor}}\ H^2_+$	$D2$	$\text{Tor}\ H_2^+$	C_{ijk}	$\text{Tor}\ H^3_+$
${C_\mu}^{mnop}$	$\widehat{\text{Tor}}\ H^4_-$	$D4$	$\text{Tor}\ H_4^-$	C_{ijklm}	$\text{Tor}\ H^5_-$

6. Torsion and discrete gauge symmetries

A particularly interesting case is that of M-theory, since it provides a unifying picture for D-brane and RR U(1) gauge symmetries.

Massive U(1) gauge symmetries spontaneoulsy broken to discrete gauge symmetries arise in this case from $\mathrm{Tor}\ H_2(\hat{\mathcal{M}}_7, \mathbb{Z}) \simeq \mathrm{Tor}\ H_4(\hat{\mathcal{M}}_7, \mathbb{Z})$

M2-branes wrapping torsional 2-cycles ⇒ 4d Aharanov-Bohm particles

M5-branes wrapping torsional 4-cycles ⇒ 4d Aharanov-Bohm strings

$$\hat{k}_\alpha{}^\beta \phi_\beta^{\mathrm{tor}} = d\omega_\alpha^{\mathrm{tor}} \qquad dA_3 = \left(\mathrm{Re}(dM^\alpha) + \hat{k}^\alpha{}_\beta A^\beta\right) \wedge \phi_\alpha^{\mathrm{tor}} + dA^\beta \wedge \omega_\beta^{\mathrm{tor}}$$

In the IIA perturbative limit they become the massive D6-brane and RR U(1)'s.

7. Conclusions

- We have considered the interplay between open and closed string U(1) gauge symmetries.

- RR U(1)'s can play a prominent role. Mixing with the hypercharge can occur either via direct kinetic mixing or via the mass terms induced by Stückelberg couplings. Interesting phenomenological implications.

- We have provided a geometric description of mass mixing in terms of the torsional homology of the CY, and developped the right tools to compute the mixing parameters in specific models.

- As a byproduct , we have provided a stringy realization of discrete gauge symmetries and 4d A-B strings and particles in terms of the torsional homology.